Special Relativity Calculation

Force $F_0 = N\, g\, m_0$ where g is the acceleration of earth gravity at 9.8 m/s$^2$ and K is a relative constant where $K = 1$ is acceleration due to earth gravity at the earth's surface. K=2 would be acceleration at 2G. $m_0$ is the rest mass.

The relativistic form of Newton's law is

$$F = \frac{dp(t)}{dt} \text{ where } p \text{ is momentum}$$

And $p = \gamma\, m_0\, a$ where a is acceleration, and $\gamma$ is Lorentz effect.

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Where v is velocity and c is the speed of light

Solving, we have

$$\frac{v(t)}{c} = \frac{F_0 t}{m_0 c} \frac{1}{\sqrt{1 + \left(\frac{F_0 t}{m_0 c}\right)^2}} = \frac{N\, g\, t}{c} \frac{1}{\sqrt{1 + \left(\frac{N\, g\, t}{c}\right)^2}}$$

And

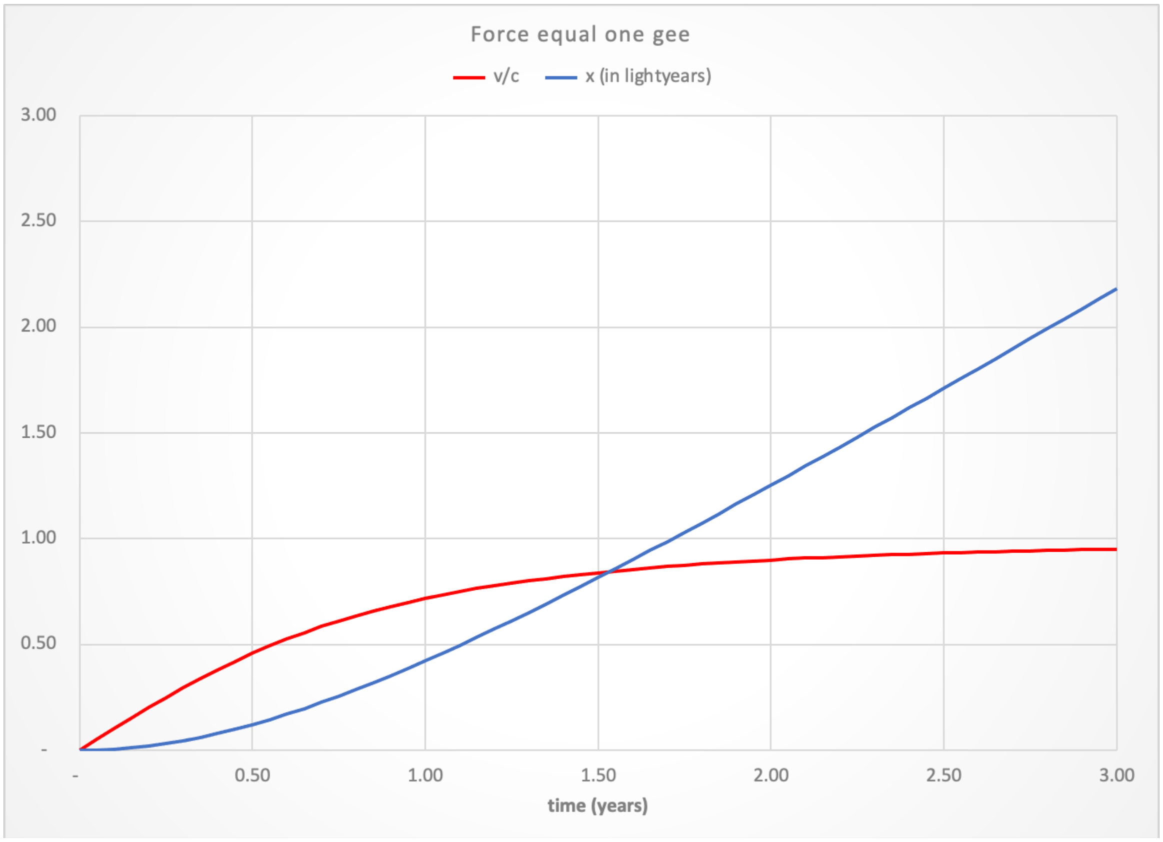
$$x(t) = \frac{c^2}{N\, g}\left[\sqrt{1 + \left(\frac{N\, g\, t}{c}\right)^2} - 1\right]$$

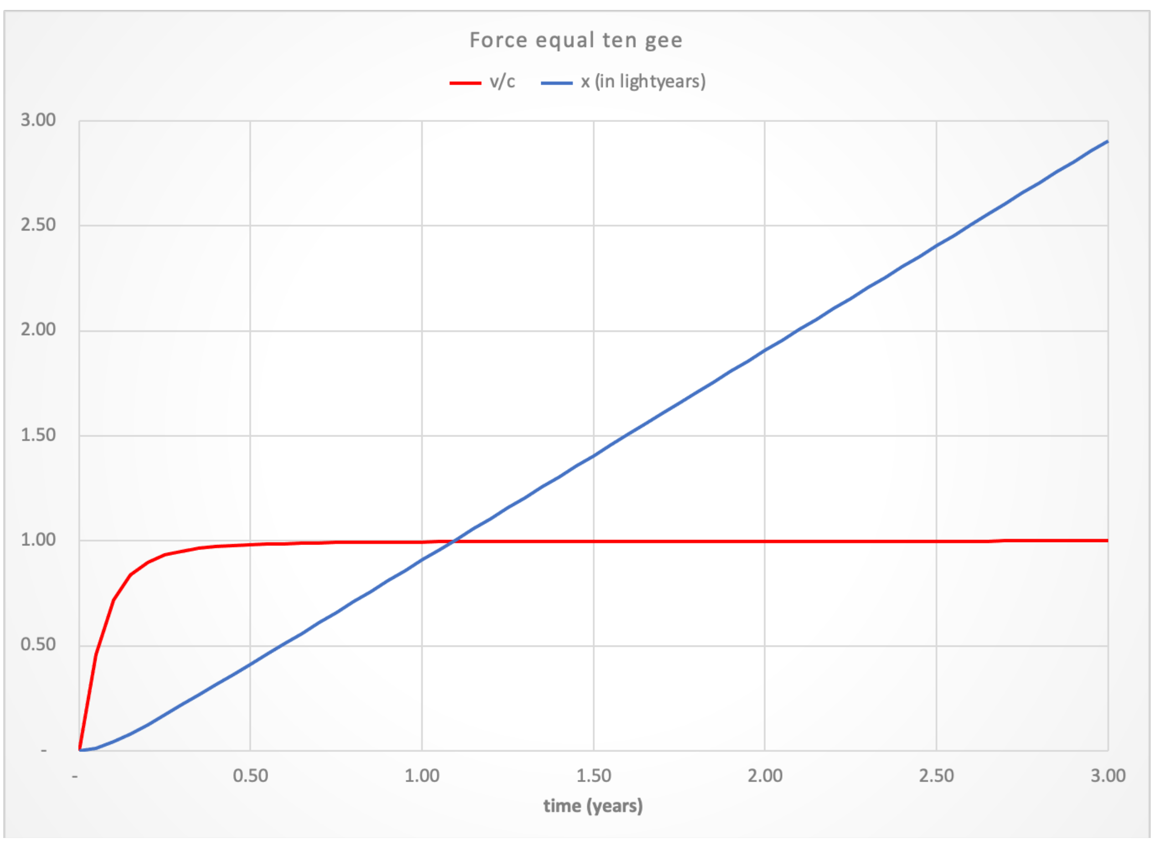
The velocity reaches relativistic speed approximately when

$$t_{relativistic} \approx \frac{c}{N\, g} \approx \frac{3 \times 10^7}{N} \text{ seconds} \approx \frac{354}{N} \text{ days}$$

Note that the acceleration is

$$a(t) = N\, g\, \frac{1}{\left(1 + \left(\frac{N\, g\, t}{c}\right)^2\right)^{3/2}}$$


Force equal one gee
v/c
x (in lightyears)
3.00
2.50
2.00
1.50
1.00
0.50
-
0.50
1.00
1.50
2.00
2.50
3.00
time (years)


Force equal ten gee
v/c
x (in lightyears)
3.00
2.50
2.00
1.50
1.00
0.50
-
0.50
1.00
1.50
2.00
2.50
3.00
time (years)

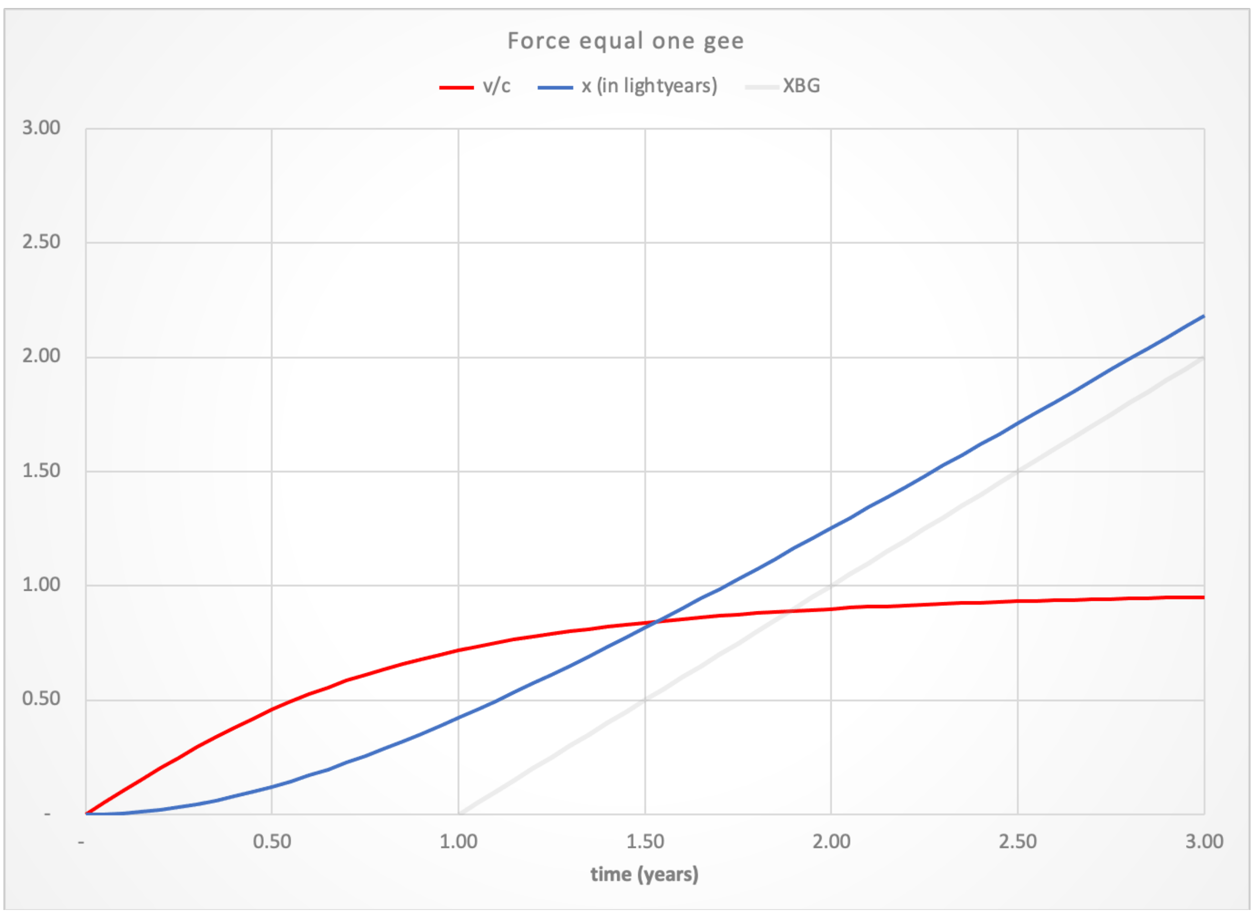
Let us assume that the Good Guys are accelerating away from earth at 1G, i.e., K=1. Can the Bad Guys catch them? If a Bad Guy starts chasing at $T_{BG}$, and moving at the speed of light, we can calculate the possible intercept time $T_x$ and distance $X_x$. where $X_x = c\,T_x$.

The time to intercept is given by

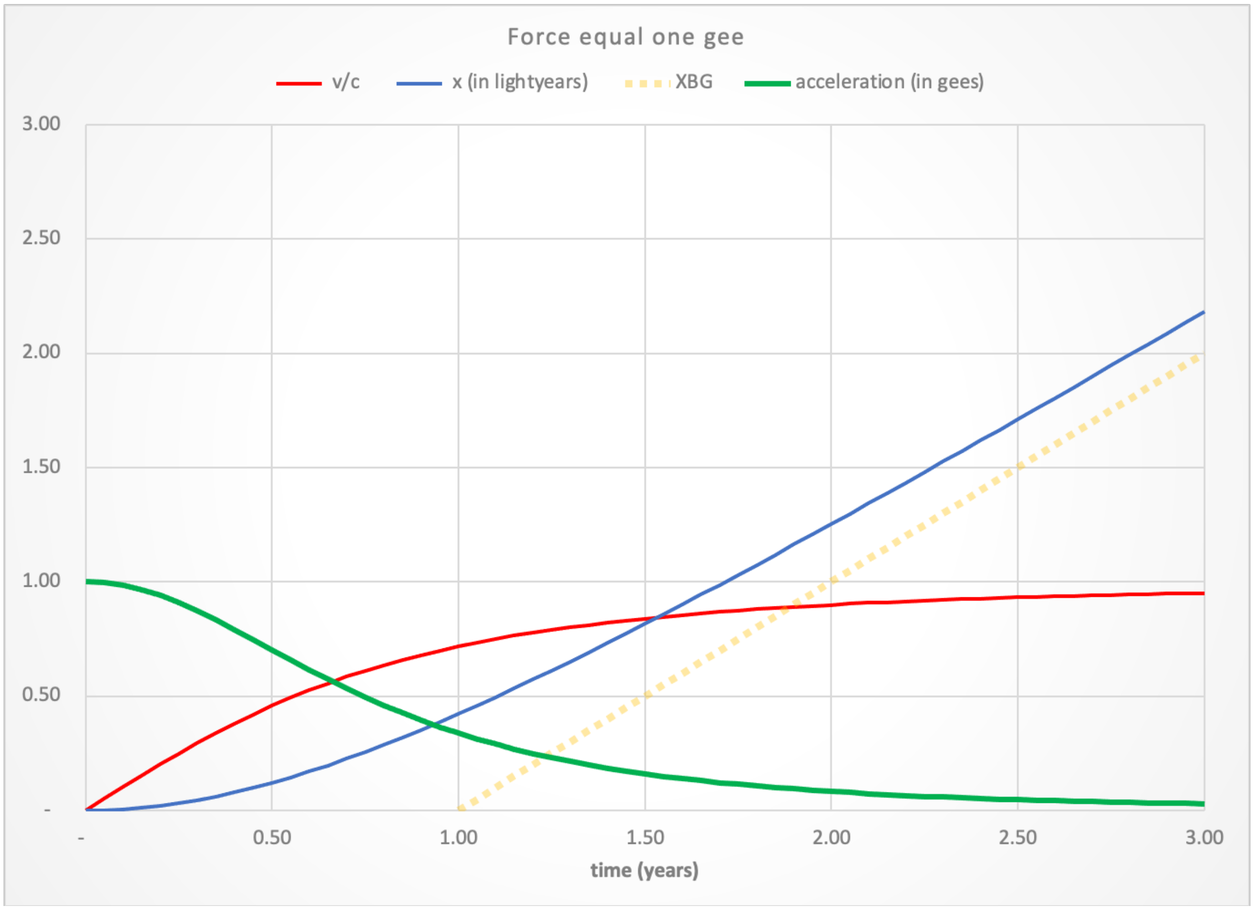
$$T_x = T_{BG} \frac{1 - \frac{N\,g\,T_{BG}}{2c}}{1 - \frac{N\,g\,T_{BG}}{c}}$$

It is fascinating that if $T_{BG} > c/(N\,g)$, the pursuer can **never** catch up, even going at the speed of light, providing that the pursued keeps up its steady acceleration. Amazingly, though the bad guys are going faster than the good guys for all time, they can never catch them provided that the good guys get enough of a head start. Highly counterintuitive.

For N = 1, if $T_{BG} > 1$ year, then pursuit will *never* succeed.




They will never get closer than 0.03 light-years. A remarkable result. Note that the volume that the Bad Guys have to search grows with the cube of time as time gets large.



Force equal one gee
v/c
x (in lightyears)
XBG
acceleration (in gees)
3.00
2.50
2.00
1.50
1.00
0.50
-
-
0.50
1.00
1.50
2.00
2.50
3.00
time (years)


Lance